

Gauge Symmetry As Symmetry Of Matrix Coordinates

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Abstract

We propose a new point of view to gauge theories based on taking the action of symmetry transformations directly on the coordinates of space. Via this approach the gauge fields are not introduced at the first step, and they can be interpreted as fluctuations around some classical solutions of the model. The new point of view is connected to the lattice formulation of gauge theories, and the parameter of non-commutativity of coordinates appears as the lattice spacing parameter. Through the statements concerning the continuum limit of lattice gauge theories, this suggestion arises that the noncommutative spaces are the natural ones to formulate gauge theories at strong coupling. Via this point of view, a close relation between the large- N limit of gauge theories and string theory can be manifested.

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Recently a great attention is appeared in formulation and studying field theories on non-commutative (NC) spaces [1, 2, 3]. Apart from abstract mathematical interests, the physical motivations in doing so have been different. One of the original motivations has been to get “finite” field theories via the intrinsic regularizations which are encoded in some of NC spaces [4, 5]. The other motivation was coming from the unification aspects of theories on NC spaces. These unification aspects has been the result of the “algebraization” of “space, geometry and their symmetries” via the approach of NC geometry [6]. Interpreting Higgs field as a gauge field in discrete direction of a two-sheet space [7] and unifying gauge theories with gravity [8, 9] are examples of this view point to NC spaces.

The other motivation comes back to the natural appearance of NC spaces in some areas of physics, and the recent one in string theory. It has been understood that string theory is involved by some kinds of non-commutativities; two examples are, 1) the coordinates of bound-states of N D-branes are presented by $N \times N$ Hermitian matrices [10], and 2) the longitudinal directions of D-branes in the presence of B-field background appear to be NC, as are seen by the ends of open strings [11, 12, 1].

As mentioned in the above, one of the motivations to formulate theories on NC spaces has been a unified treatment with the symmetries living in a space and the space itself. One of the most important symmetries in physical theories is gauge symmetry, and to be extreme in identifying the space with its symmetries is to take the action of symmetry transformations on the space. In usual gauge theories the action of the symmetry transformations is defined on the gauge fields, A^μ , but in the new picture one takes the action on space, and to be more specific on the “coordinates” of space. It will be our main strategy in presenting a new point of view to gauge theories. As it will be clear later, the main tools and view points to different subjects and discussions here are developed and coming from the D-branes of string theories [13, 14]. Here we try to reorganize the facts and discussions to present a new picture for gauge theories and see how the things should be by this approach. The action which we concern here is the Eguchi-Kawai one [15], but with a different interpretation on the configurations which are described by the action. As we will see, the new interpretation is sufficiently rich to recover some aspects of gauge theories which has been already known as maybe some disjoint facts. It will be shown that the new interpretation is related from one side to lattice formulation of gauge theories [16], and with a different representation is connected to ordinary formulation of gauge theories. In relation with lattice gauge theory the parameter of noncommutativity of coordinates appears as the lattice spacing parameter. Through the statements concerning the continuum limit of lattice gauge theories this suggestion arises that the NC spaces

are the natural ones to formulate gauge theories at the strong coupling limit. Also the model can manifest a close relation between the large- N limit of gauge theories, known to be the theory of “Feynman graphs” as the world-sheet of strings, and string theory [17].

Note: After completion of this work, I informed that lattice regularization of NC gauge theory have been constructed as a natural extension of Wilson’s lattice gauge theory. Also the relation between twisted Eguchi-Kawai model and a NC gauge theory have been studied [18].

The Model:

As mentioned in above, instead of introducing gauge fields, we define the gauge symmetry transformations directly on the generators of displacement in space, calling them “coordinates” and representing by \hat{X}^μ [3]³, and we assume to be $N \times N$ Hermitian matrices. So to describe the generators in an infinite volume these matrices should be taken for $N \rightarrow \infty$, even when they are used to formulate a finite group gauge theory. So we take the definition of the gauge transformations as:

$$\hat{X}^\mu \rightarrow \hat{X}^{\mu'} = \omega \hat{X}^\mu \omega^\dagger, \quad \mu = 1, \dots, d, \quad (1)$$

where ω is an arbitrary unitary $N \times N$ matrix (so it belongs to a group, say G). This transformation is the same of [3] but not in the infinitesimal form. On the other hand, it is the same transformation which acts on the coordinates of D-branes as $N \times N$ Hermitian matrices (see e.g. [19, 20]). So the coordinates in a space which contains the bound-states of N D-branes enjoy such a transformation. Also if from the first one chooses the matrices \hat{X}^μ s to belong to $L_\infty^2(\mathbf{R}^d) \otimes M_{n \times n}$ in the form $\hat{X}^\mu = i\partial^\mu \otimes \mathbf{1}_n + g_{YM} \mathbf{1} \otimes A^\mu$ [21], they will have the same behavior under gauge transformations such as (1). So we are not very far from the usual language of gauge theories.

Our coordinates are matrices and NC, and as usual are accompanied with a length scale which is the size that the NC effects appear. Here we show this length scale by ℓ . We define the unitary matrices:

$$U_\mu \equiv e^{i\ell \hat{X}^\mu}, \quad (2)$$

as the operators which acting on the states make the displacement ℓ . With the ideas coming from lattice gauge theory, and also reminding the role of covariant derivatives as the tools of parallel transformations, we define the objects:

$$\Omega_{\mu\nu} \equiv U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger, \quad (3)$$

³In [3] these object are called “covariant coordinates”.

with the property $\Omega_{\mu\nu}^{-1} = \Omega_{\nu\mu} = \Omega_{\mu\nu}^\dagger$. Then the action of the model we take to be:

$$S = -\frac{1}{g^2} \sum_{\mu,\nu} \text{Tr } \Omega_{\mu\nu}, \quad (4)$$

which via the Tr is invariant under the transformation (1). This action is essentially the Eguchi-Kawai one [15]. In the context of Eguchi-Kawai model the symmetry of the action is a global symmetry, i.e. the symmetry transformations on the gauge fields are space independent. But as we will see, interpreting \hat{X}^μ s as space coordinates encodes sufficiently rich structure in the model to extract gauge fields and their local symmetry transformations as the same of usual formulation of gauge theories. One may define something in analogy with the field strength as:

$$\Omega_{\mu\nu} \equiv e^{-i\ell^2 F_{\mu\nu}}, \quad (5)$$

which in small ℓ limit it takes the form:

$$F^{\mu\nu} = -i[\hat{X}^\mu, \hat{X}^\nu] + \frac{1}{2}\ell \left[\hat{X}^\mu + \hat{X}^\nu, [\hat{X}^\mu, \hat{X}^\nu] \right] + O(\ell^2). \quad (6)$$

The action in small ℓ has the form as:

$$S|_{\ell \rightarrow 0} = -\frac{1}{g^2} \sum_{\mu,\nu} \text{Tr} \left(1 - i\ell^2 F_{\mu\nu} - \frac{1}{2}\ell^4 F_{\mu\nu}^2 + \dots \right). \quad (7)$$

The linear term in $F_{\mu\nu}$ does not have contribution to the action because it is antisymmetric in $\mu\nu$ ⁴. So for small values of ℓ we have:

$$S|_{\ell \rightarrow 0} = -\frac{1}{2g^2} \ell^4 \sum_{\mu,\nu} \text{Tr} [\hat{X}^\mu, \hat{X}^\nu]^2 + \text{const. term} + O(\ell^5). \quad (8)$$

The actions (4) or (8) are actions for the matrices describing the space and its symmetries. Issues such as dynamical generation of space and its dimension, and also the gauge group via the Matrix Theory have been discussed in [22][23].

Relation To Lattice Gauge Theory (strong coupling):

The model described with action (4) has already the form of lattice gauge theory at large- N , called Eguchi-Kawai model. Here we also want to mention the connection to lattice gauge theory for finite groups. In fact the relation between NC geometry and also NC differential geometry with lattice gauge theory has already been established in the

⁴It is not true that because of Tr the linear term can be ignored. For infinite dimensional matrices one can get non-zero trace from a commutator.

previous works [24, 25]. Here we try to construct the relation explicitly. Let us have a look to the action of lattice gauge theory:

$$S_{\text{lgt}} = -\frac{1}{g^2} \sum_{\mu, \nu} \sum_{\vec{i}} \text{Tr} \left(e^{iaA_{\vec{i}}^{\mu}} e^{iaA_{\vec{i}+\mu}^{\nu}} e^{-iaA_{\vec{i}+\nu}^{\mu}} e^{-iaA_{\vec{i}}^{\nu}} \right), \quad (9)$$

with a as lattice spacing parameter and \vec{i} as a d -vector representing a site in the d -dimensional lattice. Also we have used the symbol $\vec{i} + \mu$ for $(i_1, \dots, i_{\mu} + 1, \dots, i_d)$. To get a $U(m)$ lattice gauge theory, as the first step, take the Ansatz resulted from d times block-diagonalizations of the matrices \hat{X}^{μ} s, with the size of the last block to be $m \times m$. So the action takes the form:

$$S_{\text{blocked}} = -\frac{1}{g^2} \sum_{\mu, \nu} \sum_{\vec{i}} \text{Tr} \left(e^{i\ell \hat{x}_{\vec{i}}^{\mu}} e^{i\ell \hat{x}_{\vec{i}}^{\nu}} e^{-i\ell \hat{x}_{\vec{i}}^{\mu}} e^{-i\ell \hat{x}_{\vec{i}}^{\nu}} \right), \quad (10)$$

which the index i_j in the vector \vec{i} is counting the place of a block in the j th step of block-diagonalizations. The Tr above is for the $U(m)$ structure of $\hat{x}_{\vec{i}}^{\mu}$ matrices. But this action is still different from the lattice action (9). To make the exact correspondence we should do a slight modification in one of the steps of block-diagonalizations. Firstly, take the matrix Δ as:

$$\begin{aligned} \Delta_{rs} &= \delta_{r,s-1}, & \text{for infinite size,} \\ \Delta_{rs} &= \delta_{r,s-1}, \quad \Delta_{p1} = 1, & \text{for size } p \times p, \end{aligned} \quad (11)$$

with the properties $\Delta^{-1} = \Delta^T = \Delta^{\dagger}$; so Δ is unitary. For this matrix and a diagonal matrix A we have:

$$\Delta \text{diag.}(a_1, \dots, a_{p-1}, a_p) \Delta^{-1} = \text{diag.}(a_2, \dots, a_p, a_1). \quad (12)$$

By using matrix Δ we modify the block-diagonalizations mentioned above, by requesting that in the μ th step of diagonalizations of matrix \hat{X}^{μ} , it picks up a Δ with appropriate size, as

$$\hat{X}^{\mu} \xrightarrow{\mu\text{th step}} \hat{x}^{\mu} \Delta. \quad (13)$$

So in two steps of d steps two pairs of Δ and Δ^{-1} appear around \hat{X}^{μ} and \hat{X}^{ν} matrices in the action, and this cause the appropriate shift in the blocks to obtain the action of lattice gauge theory (9). In comparison with the lattice action one sees that the parameter ℓ has appeared as the lattice spacing parameter. It means that the lattice spacing parameter is a measure for appearing NC effects [25]. Based on the lattice calculations, one can derive

the relation between parameters ℓ , the coupling constant g and the string tension K , and via this relation a statement follows that the continuum limit of lattice gauge theories are gained just at exactly zero coupling [20, 26]. So this suggestion arises that the strong coupling limit of gauge theories will find a reasonable and natural formulation in NC spaces (for more discussions on this point see [20, 19]). Also via this explicit construction this observation is done that both the structure of space (here a lattice) and also the gauge fields living in the space can be extracted from the big matrices \hat{X}^μ s. We see another example of this behavior in the relation between the model and ordinary formulation of gauge theories.

Relation To Ordinary Gauge Theory (weak coupling):

It is known that the classical action of lattice gauge theories at the small lattice parameter is equivalent with the classical action of gauge theories, so-called there, the weak coupling limit of lattice gauge theory [26]. So up to know, by taking the limit $\ell \rightarrow 0$ in the action obtained in the previous section we can get the ordinary action of gauge theories. In the following we give another presentation for this, which of course it contains the procedure of going to continuum limit, but a little implicitly. To get the ordinary gauge theory we use the techniques which have been developed in constructing D-branes from Matrix Theories [27, 28]. Here we just recall the construction and refer the reader to literature (see e.g. [29]). For large matrices one always can find a set of matrix-pairs (\hat{q}^i, \hat{p}^i) with sizes $n_i \times n_i$ s so that:

$$[\hat{q}^i, \hat{p}^j] = i\delta_{ij}\mathbf{1}_{n_i}. \quad (14)$$

The above commutator is not satisfied for finite dimensional matrices. We assume the eigenvalues of \hat{q}^i and \hat{p}^i are distributed uniformly in the interval $[0, \sqrt{2\pi n_i}]$. To get a $U(m)$ gauge theory one can break the matrices \hat{X}^μ s with size N to matrices with sizes n_i s and m such that: $N = m \cdot n_1 n_2 \dots n_{d/2}$ when d is even, and $N = m \cdot n_1 n_2 \dots n_{(d+1)/2}$ for d odd, with the condition $N, n_i \rightarrow \infty$ and m finite. On the other hand, it is easy to see that matrices in the form:

$$\begin{aligned} \ell^2 \hat{X}_{\text{cl}}^{2i-1} &= \underbrace{\mathbf{1}_{n_1} \otimes \dots}_{i-1} \frac{\hat{q}^i L_i}{\sqrt{2\pi n_i}} \otimes \dots \otimes \mathbf{1}_{n_{d/2}} \otimes \mathbf{1}_m, \\ \ell^2 \hat{X}_{\text{cl}}^{2i} &= \underbrace{\mathbf{1}_{n_1} \otimes \dots}_{i-1} \frac{\hat{p}^i L_{i+1}}{\sqrt{2\pi n_i}} \otimes \dots \otimes \mathbf{1}_{n_{d/2}} \otimes \mathbf{1}_m, \quad i = 1, \dots, d/2, \end{aligned} \quad (15)$$

for even d , and with an extra one as:

$$\ell^2 \hat{X}_{\text{cl}}^d = \underbrace{\mathbf{1}_{n_1} \otimes \cdots}_{\frac{d-1}{2}} \frac{\hat{q}^{\frac{d+1}{2}} L_d}{\sqrt{2\pi n_{\frac{d+1}{2}}}} \otimes \mathbf{1}_m, \quad (16)$$

for odd d , solve the equations of motion derived from the action. Here L_i s have the interpretation as the large radii of compactifications [27, 28]. By the equations of motion for n_i s one obtains [28]:

$$\frac{L_i L_{i+1}}{2\pi n_i} \sim \ell^2. \quad (17)$$

By admitting fluctuations around classical solutions, one can write:

$$\hat{X}^\mu = \hat{X}_{\text{cl}}^\mu + g_{YM} A^\mu, \quad (18)$$

with A_μ s as $N \times N$ Hermitian matrices and functions of (\hat{q}^i, \hat{p}^i) matrices, also with the same structure of matrices \hat{X}_μ s. By inserting \hat{X}_μ s and expanding the action in the $\ell \rightarrow 0$ limit up to second order of fluctuations, and with identifications [29, 28, 27]:

$$\begin{aligned} [\hat{p}_i, *] &\sim i\partial_{2i-1}*, \\ [\hat{q}_i, *] &\sim i\partial_{2i}*, \\ \text{Tr}(\cdots) &\rightarrow \int d^d x (\cdots), \end{aligned} \quad (19)$$

one recovers the ordinary action for $U(m)$ gauge theory. The coupling constant of the resulted gauge theory is found to be $g_{YM}^2 \sim \ell^{d-4} g^2$, which in the limit of small ℓ and for $d \geq 4$ the theory corresponds to the weak coupling limit.

Large- N Gauge Theory And String Theory:

It is known that in a diagrammatic representation, the partition function of a gauge theory at large- N is given by $(\frac{1}{N})^{\text{genus}}$ expansion, with genus to be that of the “big” Feynman graphs of the theory. Also it is shown that the density of “holes” (quark loops) in the graphs goes to zero with $\frac{1}{N}$. So in the extreme large- N limit the theory is described by smooth graphs. By interpreting $\frac{1}{N}$ as the coupling constant of a string theory, the expansion mentioned above takes the form of the standard string perturbation one [17]. All of the features mentioned here can be described by the point of view proposed in this work. Firstly, at large- N the action can take the form of that of free strings. We are thinking about smooth strings, so we take $\ell \rightarrow 0$ and $N \rightarrow \infty$. So the action becomes:

$$S|_{\ell \rightarrow 0} = -\frac{1}{2\ell^4 g^2} \sum_{\mu, \nu} \text{Tr} [\hat{X}^\mu, \hat{X}^\nu]^2 - \frac{1}{g^2} \text{Tr} \mathbf{1}_N, \quad (20)$$

which we have applied the replacement $\hat{X}^\mu \rightarrow \hat{X}^\mu/\ell^2$ and so the new \hat{X}^μ has the length dimension. To get the free strings one use the map between the matrix variables (\hat{q}, \hat{p}) and continuous phase space variables (σ_1, σ_2) as [28, 27, 30]:

$$\begin{aligned} \text{Tr}(\cdots) &\rightarrow \int d^2\sigma \sqrt{\det g_{rs}}(\cdots), \\ [A, B] &\rightarrow \{A, B\}_{\text{PB}}, \quad [\hat{q}, \hat{p}] = i \rightarrow \{\sigma_1, \sigma_2\}_{\text{PB}} = 1/\sqrt{\det g_{rs}}, \\ [\hat{p}, *] &\rightarrow i\partial_1*, \quad [\hat{q}, *] \rightarrow i\partial_2*, \end{aligned} \tag{21}$$

with the definition for Poisson bracket as $\{A, B\}_{\text{PB}} = \frac{1}{\sqrt{\det g_{rs}}} \epsilon^{rs} \partial_r A \partial_s B$, ($r, s = 1, 2$). By these replacements one gets the action of free strings in the Schild form [31, 28]. Also by solving the equation of motion for $\sqrt{\det g_{rs}}$ and inserting the solution in the action one can obtain the Nambu-Goto action.

The issue of interaction is more subtle, and also has been approached previously [32]. It is shown that the $\frac{1}{N}$ expansion for this action corresponds to perturbation theory of strings by reproducing the light-cone string field theory through the Schwinger-Dyson equations.

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